# Frequently Asked Questions about the Finite Element Method

1. What is the finite element method (FEM)?

The FEM is a novel numerical method used to solve ordinary and partial differential equations. The method is based on the integration of the terms in the equation to be solved, in lieu of point discretization schemes like the finite difference method. The FEM utilizes the method of weighted residuals and integration by parts (Green-Gauss Theorem) to reduce second order derivatives to first order terms. The FEM has been used to solve a wide range of problems, and permits physical domains to be modeled directly using unstructured meshes typically based upon triangles or quadrilaterals in 2-D and tetrahedrons or hexahedrals in 3-D. The solution domain is discretized into individual elements – these elements are operated upon individually and then solved globally using matrix solution techniques.

2. What is the history of the FEM?

Early work on numerical solution of boundary-valued problems can be traced to the use of finite difference schemes; Southwell used such methods in his book published in the mid 1940's. The beginnings of the finite element method actually stem from these early numerical methods and the frustration associated with attempting to use finite difference methods on more difficult, geometrically irregular problems. Beginning in the mid 1950s, efforts to solve continuum problems in elasticity using small, discrete "elements" to describe the overall behavior of simple elastic bars began to appear, and such techniques were initially applied to the aircraft industry. Actual coining of the term "finite element" appeared in a paper by Clough in 1960. The early use of finite elements lay in the application to structural-related problems. However, others soon recognized the versatility of the method and its underlying rich mathematical basis for application in non-structural areas. Since these early works, rapid growth in usage of the method has continued since the mid 1970s. Numerous articles and texts have been published, and new applications appear routinely in the literature.

3. What is the Method of Weighted Residuals, i.e., Galerkin's Method?

The underlying mathematical basis of the finite element method first lies with the classical Rayleigh-Ritz and variational calculus procedures. These theories provided the reasons why the finite element method worked well for the class of problems in which variational statements could be obtained (e.g., linear diffusion type problems). However, as interest expanded in applying the finite element method to more types of problems, the use of classical theory to describe such problems became limited and could not be applied, e.g., fluid-related problems. Extension of the mathematical basis to non-linear and non-structural problems was achieved through the method of weighted residuals (MWR), originally conceived by Galerkin in the early 20th century. The MWR was found to provide the ideal theoretical basis for a much wider basis of problems as opposed to the Rayleigh-Ritz method. Basically, the method requires the governing differential equation to be multiplied by a set of predetermined weights and the resulting product integrated over space; this integral is required to vanish. Technically, Galerkin's method is a subset of the general MWR procedure, since various types of weights can be utilized; in the case of Galerkin's method, the weights are chosen to be the same as the functions used to define the unknown variables. Most practitioners of the finite element method now employ Galerkin's method to establish the approximations to the governing equations.

4. Why should one use finite elements?

The versatility of the FEM, along with its rich mathematical formulation and robustness makes it an ideal numerical method for a wide range of problems. The ability to model complex geometries using unstructured meshes and employing elements that can be individually tagged makes the method unique. The ease of implementing boundary conditions as well as being able to use a wide family of element types is a definite advantage of the scheme over other methods. In addition, the FEM can be shown to stem from properly-posed functional minimization principles.

5. Can the FEM handle a wide range of problems, i.e., solve general PDEs?

While the FEM was initially developed to solve diffusion type problems, i.e., stress-strain equations or heat conduction, advances over the past several decades have enabled the FEM to solve advection-dominated problems, including incompressible as well as compressible fluid flow. Modifications to the basic procedure (utilizing forms of upwinding for advection, i.e., Petrov-Galerkin and adaptive meshes) allow general advection-diffusion transport equations to be accurately solved for a wide range of problems.

6. What is the advantage of the FEM over finite difference (FDM) and finite volume (FVM) methods?

The major advantages of the FEM over FDM and FVM are its built-in abilities to handle unstructured meshes, a rich family of element choices, and natural handling of boundary conditions (especially flux relations). The FDM is generally restricted to simple geometries in which an orthogonal grid can be constructed; for irregular geometries, a global transformation of the governing equations (e.g., boundary fitted coordinates) must be made to create an orthogonal computational domain. Likewise, implementation of boundary conditions in FDM can be cumbersome. The FVM is an integral approach (typically with limits -0.5 to 0.5) similar to the FEM, with volumes being used instead of elements. The divergence theorem is used to establish the final equation set. Solutions are obtained at volume faces, vertices, or volume centers – some methods employ staggered grids. While FVM can handle irregular domains using unstructured grids (stemming from the FEM), the required averaging over the volume limits the method to second order spatial accuracy.

7. Is there any connection between the FEM and the boundary element method (BEM)?

In the BEM, one reduces the order of the problem by one, i.e., a two-dimensional domain is reduced to a line integral – a three-dimensional domain becomes a two-dimensional surface. The BEM only requires the discretization of the boundaries of the problem domain – no internal meshing is required, as in the FDM, FVM, and FEM schemes. The BEM requires two applications of the Green-Gauss Theorem (versus one in the FEM and employing Galerkin's Method). The method is ideal for handling irregular shapes and setting boundaries that may extent to (near) infinity. One can place interior nodes within the BEM to obtain internal values easily. The BEM works quite effectively for linear differential equations – principally elliptic equations. However, if one desires to solve nonlinear advection-diffusion transport equations, the method becomes very cumbersome and computationally demanding – BEM matrices are dense, and do not readily permit efficient, sparse matrix solvers to be used as in the FEM.

8. What is adaptivity, i.e., h-, p-, r-, and hp-adaptation?

Adaptivity is an active research area involving either remeshing or increased interpolation order during the solution process. The method is particularly effective in fluid flow, heat transfer, and structural analysis. The use of mesh refinement has been especially effective in aerodynamic simulations for accurately capturing shock locations in compressible flow. Generally, there are two types of adaptation: h-adaptation (mesh refinement), where the element size varies while the orders of the shape functions are kept constant; p-adaptation, where the element size is constant while the orders of the shape functions are increased (linear, quadratic, cubic, etc.). Adaptive remeshing (known as r-adaptation) employs a spring analogy to redistribute the nodes in an existing mesh - no new nodes are added; the accuracy of the solution is limited by the initial number of nodes and elements. In mesh refinement (h-adaptation), individual elements are subdivided without altering their original position. The use of hp-adaptation includes both h- and p-adaptation strategies and produces exponential convergence rates. Both mesh refinement and adaptive remeshing are now routinely used in many commercial codes. A spectral element is a special class of FEM that uses a series of orthogonal basis functions whereby the unknown terms are solved at selected spectral nodes; the method is stable and highly accurate, but can become time consuming.

9. How difficult is it to write a FEM program?

Writing a FEM code is not terribly difficult, especially if one develops the code utilizing a general set of subroutines, e.g., input data, integration, assembly, boundary conditions, output, etc. About 90% of a FEM program is generic, which is fairly common among most FEM codes – they tend to use similar matrix solvers, quadrature rules, and matrix assembly procedures; I/O is usually the major difference among commercial FEM codes – some are easy, and some are not so easy to learn and use. A source listing of the FORTRAN codes can be found in the FORTRAN file folder; flow charts can be obtained from the authors. Likewise, MATLAB and MathCad files are also available. One of the best commercial packages now on the market is COMSOL, which also allows users to write their own solver packages and PDEs.

10. Are there any recommended commercial FEM packages that are versatile in handling a wide range of problems?

Any of the well known and widely versatile FEM codes now on the market are good – it just depends on how comfortable the user is with the I/O part of the program. COMSOL, as mentioned before, is quite easy and very versatile – handling a wide range of problem classes including fluid flow (with turbulence), heat transfer, structural analysis, electrodynamics, and general PDEs including species transport, chemical reactions, and groundwater/porous media flows.

11. Any suggested web sites for FEM?

There are several recommended web sites:

a. www.wiley.com/go/bhatti
b. http://dehesa.freeshell.org/FSEM
c. http://www.ncacm.unlv.edu
d. http://www.cfd-online.com/Resources/topics.html#fem

12. How long does it take for me to be able to use a FEM program?

Some programs allow you to solve problems fairly quickly. It is always highly recommended that work out the example problems generally provided by most commercial software. COMSOL, ANSYS, ALGOR, and NASTRAN all run on PCs.

13. Why would I want to use a FEM program?

The versatility, ease of data input, and solution accuracy make the FEM one of the best numerical methods for solving engineering problems. FEM programs are the backbone of structural analyses, and are becoming more widely accepted for problems in which geometries are complex.

14. Is this a method that will soon become obsolete?

The recent introduction of BEM and meshless methods would appear to indicate the eventual obsolescence of the FEM. However, these newer methods are still years away from being developed to the point of wide spread applicability found in FEM. The FEM will be around for many years to come. Recent advances with the inclusion of spectral schemes and adaptivity make it especially attractive now.

15. How expensive is a FEM code?

FEM codes range from those that can be found for free on the web to others costing many thousands of dollars. Those that run on PCs are generally inexpensive, yet provide powerful tools for solving a number of large scale problems.

16. What kind of hardware do I need to run a FEM code?

A PC with a sufficiently fast processor, at least 256MB RAM, and at least 20GB of hard disk will permit many problems to be solved that once could only be run on mainframe computers. A suggested PC level for major FEM calculations is one with 1 GB RAM, 60 GB hard disk, and running with Pentium 4/3.2 GHz or better processors would provide more that adequate capabilities. The state-of-the-art in PC hardware is improving constantly; in a few years, even these suggested requirements will seem obsolete.